CORRIGENDUM TO "LINEAR RESPONSE FORMULA FOR PIECEWISE EXPANDING UNIMODAL MAPS," NONLINEARITY, 21 (2008) 677–711

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1. Theorem 7.1

The last claim of Theorem 7.1 should read: "If the postcritical orbit of f_0 is dense in $[c_2, c_1]$ then there exist $\varphi \in C^\infty(I)$ and a sequence $t_n \to 0$ so that c is not periodic under any f_{t_n}, and so that we have

$$\lim_{n\to\infty} |t_n^{-1}(\int \varphi\rho_{t_n} - \int \varphi\rho_0 \, dx)| \to \infty \, ."$$

Proof if the orbit of c is dense. We have $\mathcal{E}_0 = \mathcal{F}_0 = \mathrm{id}$ and, using (94), we can consider $\mathcal{P}_t$ as in the case when the orbit is infinite but not dense.

Let x_0 be the fixed point of f which lies in the interior of I, and assume that $\varphi(x_0) = 1$ and $\int \varphi \, d\mu = 0$. Since the postcritical orbit is dense, for any $\delta > 0$ there exists $j_0(\delta) \geq 1$ so that $d(c_{j_0}, x_0) < \delta$. Clearly, $\lim_{\delta\to 0} j_0 = \infty$. Put $\Lambda_f = \sup |f'|$. If $\delta\Lambda_f^m \leq 1/2$ for some large m then for all $j_0 \leq n \leq j_0 + m$ we have

$$\sum_{k=0}^{n-j_0} |c_{j_0+k} - f^k(x_0)| \leq \delta \sum_{k=0}^{n-j_0} \Lambda_f^k \leq \frac{\delta\Lambda_f^{n-j_0}}{1 - 1/\Lambda_f} \, ,$$

and thus

$$\begin{aligned} |\sum_{k=1}^{n} \varphi(c_k)| &\geq |\sum_{k=0}^{n-j_0} \varphi(f^k(x_0))| - \sum_{k=j_0}^{n} |\varphi(c_k) - \varphi(f^{k-j_0}(x_0))| - |\sum_{k=1}^{j_0-1} \varphi(c_k)| \\ &\geq (n - j_0 + 1) - \frac{1}{2(1 - 1/\Lambda_f)} \sup |\varphi'| - |\sum_{k=1}^{j_0-1} \varphi(c_k)| \, . \qquad (\star) \end{aligned}$$

Let $t_n \to 0$ be a sequence of non periodic parameters and let $M(t_n)$ be defined by (92). Now,

$$\sum_{k=1}^{M(t_n)} \varphi(c_k) \sum_{j=1}^{k} \frac{X(c_j)}{(f^{j-1})'(c_1)} = \mathcal{J}(f, X) \sum_{k=1}^{M(t_n)} \varphi(c_k) - \sum_{k=1}^{M(t_n)} \varphi(c_k) \sum_{j=k+1}^{\infty} \frac{X(c_j)}{(f^{j-1})'(c_1)}$$

and

$$\begin{aligned} |\sum_{k=1}^{M(t_n)} \varphi(c_k) \sum_{j=k+1}^{\infty} \frac{X(c_j)}{(f^{j-1})'(c_1)}| &\leq \sup |X| \sup |\varphi| \sum_{k=1}^{M(t_n)} \frac{(\inf |f'|)^{-k}}{1 - 1/\inf |f'|} \\ &\leq \sup |X| \sup |\varphi| \frac{(\inf |f'|)^{-1}}{(1 - 1/\inf |f'|)^2} \end{aligned}$$

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Thus, for arbitrarily large n, recalling also $|C_n| \le \widehat{C}$ from the previous cases,

$$\left|\sum_{k=1}^{M(t_n)} \frac{s_{1,t_n}}{(f_{t_n}^{k-1})'(c_1)} \int \varphi \frac{H_{c_{k,t_n}} - H_{c_k}}{t_n} dx\right| = \left|C_n + s_1 \sum_{k=1}^{M(t_n)} \varphi(c_k) \sum_{j=1}^{k} \frac{X(c_j)}{(f^{j-1})'(c_1)}\right|$$

$$\ge |\sum_{k=1}^{M(t_n)} \varphi(c_k)||\mathcal{J}(f,X)| - \widehat{C} - \sup|\varphi| \sup|X| \frac{\inf|f'|^{-1}}{(1-1/\inf|f'|)^2}\,.$$

Assume now for a contradiction that for any sequence $t_n \to 0$ as above we have $|\int \varphi d\mu_{t_n}| \le A|t_n|$ for some $A < \infty$ and all large enough n. Then, for all large enough n, we would have

$$|\sum_{k=1}^{M(t_n)} \varphi(c_k)| \le \frac{A + \widetilde{C}(f_t,\varphi)}{|\mathcal{J}(f,X)|} =: D\,. \qquad (\star\star)$$

To end the proof we shall find sequences t_n so that the above estimate gives a contradiction. (Note that φ cannot be a coboundary since $\varphi(x_0) = 1$.)

For $m \ge 1$, let $\delta(m) > 0$ be so that $\delta \Lambda_f^m < 1/2$. Next, take $j_0 = j_0(\delta(m)) \ge 1$ so that $d(c_{j_0}, x_0) < \delta$. Then, letting $J(m) \ge 1$ be minimum for the property $j_0(\delta(m + J(m))) - 1 > j_0(\delta(m)) + m$, we have

$$j_0(\delta(m)) - 1 < j_0(\delta(m)) + m < j_0(\delta(m + J(m))) - 1 < \dots\,,$$

and this defines a sequence, denoted $L(n)$, so that $L(n) \to \infty$ as $n \to \infty$. We claim that we can choose the sequence $t_n \to 0$ of nonperiodic parameters so that for all large enough n we have $M(t_n) = L(n)$. Indeed, by the definition of $M(t)$, and since $\inf|f'| > 1$, there is a sequence $0 < \tau_L < \tau_{L-1}$, $L \ge 1$, with $\tau_L \to 0$ as $L \to \infty$, so that for any $t \in [\tau_L, \tau_{L-1})$ we have $M(t) = L$. Thus, since the set of non periodic parameters is dense (see [1, Cor. 4.1, item A]), there is a sequence of non periodic parameters $t_n \to 0$ so that $M(t_n) = L(n)$.

Then, recalling $(\star)$,

$$|\sum_{k=1}^{j_0(\delta(m))+m-1} \varphi(c_k)| \ge m - \frac{1}{2(1-1/\Lambda_f)} \sup|\varphi'| - |\sum_{k=1}^{j_0(\delta(m))-1} \varphi(c_k)|$$

$$\ge m - \frac{1}{2(1-1/\Lambda_f)} \sup|\varphi'| - D\,.$$

The rightmost lower bound clearly diverges as $m \to \infty$, giving the desired contradiction with $(\star\star)$. $\square$

2. Typographical errors

We use this opportunity to correct two minor typographical errors:

In the Proof of Theorem 7.1, if the orbit of c is infinite but not dense, "We now consider the first term in (95)" on p. 706, line 6, should read "We now consider the second term in (95)."

In the beginning of §3.3, functions in $\widetilde{BV}$ are supported in $(-\infty, b]$, not $[a, b]$.

References

[1] V. Baladi and D. Smania, *Smooth deformations of piecewise expanding unimodal maps,* Discrete and Continuous Dynamical Systems Series A **23** (2009) 685–703.